SOLUTIONS TO CONCEPTS CHAPTER – 8

1.
$$M = m_c + m_b = 90 \text{kg}$$

$$u = 6 \text{ km/h} = 1.666 \text{ m/sec}$$

$$v = 12 \text{ km/h} = 3.333 \text{ m/sec}$$

Increase in K.E. =
$$\frac{1}{2}$$
 Mv² – $\frac{1}{2}$ Mu²

=
$$\frac{1}{2}$$
 90 × (3.333)² – $\frac{1}{2}$ × 90 × (1.66)² = 494.5 – 124.6 = 374.8 \approx 375 J

2.
$$m_b = 2 \text{ kg}$$
.

$$u = 10 \text{ m/sec}$$

$$a = 3 \text{ m/aec}^2$$

$$t = 5 sec$$

$$v = u + at = 10 + 3 I 5 = 25 m/sec.$$

∴ F.K.E =
$$\frac{1}{2}$$
 mv² = $\frac{1}{2}$ × 2 × 625 = 625 J.

3. F = 100 N

$$S = 4m$$
, $\theta = 0^{\circ}$

$$\omega = \vec{\mathsf{F}}.\vec{\mathsf{S}} \, = 100 \times 4 = 400 \, \mathsf{J}$$

4.
$$m = 5 \text{ kg}$$

$$\theta$$
 = 30°

$$S = 10 \text{ m}$$

$$F = mg$$

So, work done by the force of gravity

$$\omega = mgh = 5 \times 9.8 \times 5 = 245 J$$

5. F= 2.50N, S= 2.5m, m=15g=0.015kg.

So, w = F × S
$$\Rightarrow$$
 a = $\frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3}$ m/s²

=F × S cos 0° (acting along the same line)

$$= 2.5 \times 2.5 = 6.25 J_{\bullet}$$

Let the velocity of the body at b = U. Applying work-energy principle $\frac{1}{2}$ mv² – 0 = 6.25

$$\Rightarrow$$
 V = $\sqrt{\frac{6.25 \times 2}{0.015}}$ = 28.86 m/sec.

So, time taken to travel from A to B.

$$\Rightarrow t = \frac{v - u}{a} = \frac{28.86 \times 3}{500}$$

:. Average power =
$$\frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3} = 36.1$$

6. Given

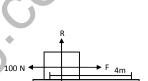
$$\vec{r}_1 = 2\hat{i} + 3\hat{j}$$

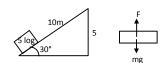
$$r_2 = 3\hat{i} + 2\hat{j}$$

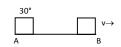
So, displacement vector is given by,

$$\vec{r} = \vec{r}_1 - \vec{r}_2 \implies \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$$









- So, work done = $\vec{F} \times \vec{s} = 5 \times 1 + 5(-1) = 0$
- 7. $m_b = 2kg$, s = 40m, $a = 0.5m/sec^2$
 - So, force applied by the man on the box

..(2)

$$F = m_b a = 2 \times (0.5) = 1 N$$

$$\omega$$
 = FS = 1 × 40 = 40 J

8. Given that F = a + bx

Where a and b are constants.

So, work done by this force during this force during the displacement x = 0 and x = d is given by

W =
$$\int_0^d F dx = \int_0^d (a + bx) dx = ax + (bx^2/2) = [a + \frac{1}{2}bd] d$$

9. $m_b = 250g = .250 \text{ kg}$

$$\theta$$
 = 37°, S = 1m.

Frictional force $f = \mu R$

mg sin
$$\theta = \mu R$$
 ...(1)

mg
$$\cos \theta$$

so, work done against μ R = μ RS cos 0° = mg sin θ S = 0.250 × 9.8 × 0.60 × 1 = 1.5 J



- 10. $a = \frac{F}{2(M+m)}$ (given)
 - a) from fig (1)

 $ma = \mu_k R_1$ and $R_1 = mg$

$$\Rightarrow \mu = \frac{ma}{R_1} = \frac{F}{2(M+m)g}$$

c) Work done w = fs

b) Frictional force acting on the smaller block $f = \mu R = \frac{F}{2(M+m)g} \times mg = \frac{m \times F}{2(M+m)g}$



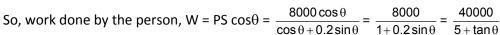
11. Weight = 2000 N, S = 20m, $\mu = 0.2$

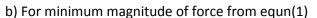
a) R + Psin
$$\theta$$
 - 2000 = 0 ...(1

$$P \cos\theta - 0.2 R = 0$$
 ...(2)

From (1) and (2) $P \cos\theta - 0.2 (2000 - P \sin\theta) = 0$

$$P = \frac{400}{\cos\theta + 0.2\sin\theta} \qquad ...(3)$$



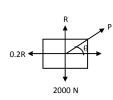


$$d/d\theta (\cos \theta + 0.2 \sin \theta) = 0 \Rightarrow \tan \theta = 0.2$$

putting the value in equn (3)

$$W = \frac{40000}{5 + \tan \theta} = \frac{40000}{(5.2)} = 7690 \text{ J}$$

12.
$$w = 100 \text{ N}, \ \theta = 37^{\circ}, \ s = 2\text{m}$$



Force F= mg sin $37^{\circ} = 100 \times 0.60 = 60 \text{ N}$

So, work done, when the force is parallel to incline.

$$w = Fs \cos \theta = 60 \times 2 \times \cos \theta = 120 J$$

In ΔABC AB= 2m

$$CB = 37^{\circ}$$

so,
$$h = C = 1m$$

: work done when the force in horizontal direction

$$W = mgh = 100 \times 1.2 = 120 J$$

13.
$$m = 500 \text{ kg}$$
, $s = 25 \text{m}$, $u = 72 \text{km/h} = 20 \text{ m/s}$,

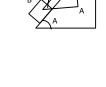
$$(-a) = \frac{v^2 - u^2}{2S} \Rightarrow a = \frac{400}{50} = 8 \text{m/sec}^2$$

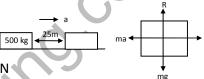
Frictional force $f = ma = 500 \times 8 = 4000 \text{ N}$

14.
$$m = 500 \text{ kg}$$
, $u = 0$, $v = 72 \text{ km/h} = 20 \text{m/s}$

$$a = \frac{v^2 - u^2}{2s} = \frac{400}{50} = 8m/sec^2$$

force needed to accelerate the car $F = ma = 500 \times 8 = 4000 \text{ N}$





v = 0

v=20 m/s

15. Given, $v = a \sqrt{x}$ (uniformly accelerated motion)

displacement s = d - 0 = d

putting
$$x = 0$$
, $v_1 = 0$

putting
$$x = d$$
, $v_2 = a \sqrt{d}$

$$a = \frac{{v_2}^2 - {u_2}^2}{2s} = \frac{a^2d}{2d} = \frac{a^2}{2}$$

force f = ma =
$$\frac{\text{ma}^2}{2}$$

work done w = FS
$$\cos \theta = \frac{\text{ma}^2}{2} \times d = \frac{\text{ma}^2 d}{2}$$

16. a) m = 2kg,
$$\theta = 37^{\circ}$$
, F = 20 N

From the free body diagram

$$F = (2g \sin \theta) + ma \Rightarrow a = (20 - 20 \sin \theta)/s = 4m/sec^2$$

$$S = ut + \frac{1}{2}at^{2}$$
 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done $w = Fs = 20 \times 2 = 40 J$

b) If
$$W = 40 J$$

$$S = \frac{W}{F} = \frac{40}{20}$$

$$h = 2 \sin 37^{\circ} = 1.2 m$$

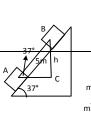
So, work done W =
$$-mgh = -20 \times 1.2 = -24 J$$

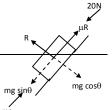
c)
$$v = u + at = 4 \times 10 = 40 \text{ m/sec}$$

So, K.E. =
$$\frac{1}{2}$$
 mv² = $\frac{1}{2}$ × 2 × 16 = 16 J

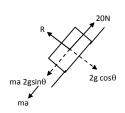
17. m = 2kg,
$$\theta$$
 = 37°, F = 20 N, a = 10 m/sec²

So,
$$s = ut + \frac{1}{2} at^2 = 5m$$









Work done by the applied force w = FS $\cos 0^{\circ} = 20 \times 5 = 100 \text{ J}$

b) BC (h) =
$$5 \sin 37^{\circ} = 3m$$

So, work done by the weight W = mgh = $2 \times 10 \times 3 = 60$ J

c) So, frictional force
$$f = mg \sin\theta$$

work done by the frictional forces w = fs $\cos 0^{\circ}$ = (mg $\sin \theta$) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 250 g = 0.250 kg,

$$u = 40 \text{ cm/sec} = 0.4 \text{m/sec}$$

$$\mu = 0.1$$
, v=0

Here, μ R = ma {where, a = deceleration}

$$a = \frac{\mu R}{m} = \frac{\mu mg}{m} = \mu g = 0.1 \times 9.8 = 0.98 \text{ m/sec}^2$$

$$S = \frac{v^2 - u^2}{2a} = 0.082m = 8.2 \text{ cm}$$

Again, work done against friction is given by

$$- w = \mu RS \cos \theta$$

$$= 0.1 \times 2.5 \times 0.082 \times 1 (\theta = 0^{\circ}) = 0.02 \text{ J}$$

$$\Rightarrow$$
 W = -0.02 J

19. h = 50m, m =
$$1.8 \times 10^5$$
 kg/hr, P = 100 watt,

P.E. = mgh =
$$1.8 \times 10^5 \times 9.8 \times 50 = 882 \times 10^5 \text{ J/hr}$$

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441×10^5 J/hr

So, power in watt (J/sec) is given by =
$$\frac{441 \times 10^3}{3600}$$

∴ number of 100 W lamps, that can be lit
$$\frac{441 \times 10^5}{3600 \times 100}$$
 = 122.5 ≈122

20.
$$m = 6kg$$
, $h = 2m$

P.E. at a height '2m' = mgh =
$$6 \times (9.8) \times 2 = 117.6 \text{ J}$$

P.E. at floor
$$= 0$$

Loss in P.E. =
$$117.6 - 0 = 117.6 \text{ J} \approx 118 \text{ J}$$

21.
$$h = 40m$$
, $u = 50 \text{ m/sec}$

Let the speed be 'v' when it strikes the ground.

Applying law of conservation of energy

$$mgh + \frac{1}{2} mu^2 = \frac{1}{2} mv^2$$

$$\Rightarrow$$
 10 × 40 + (1/2) × 2500 = ½ $v^2 \Rightarrow v^2 = 3300 \Rightarrow v = 57.4$ m/sec \approx 58 m/sec

$$p = \frac{w}{t}$$
, Work $w = pt = 460 \times 117.56 J$

Again, W = FS =
$$\frac{460 \times 117.56}{200}$$
 = 270.3 N \approx 270 N

23.
$$S = 100 \text{ m}$$
, $t = 10.54 \text{ sec}$, $m = 50 \text{ kg}$

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

- a) Speed v = S/t = 9.487 e/s
- So, K.E. = $\frac{1}{2}$ mv² = 2250 J
- b) Weight = mg = 490 J
- given R = mg / 10 = 49 J
- so, work done against resistance $W_F = -RS = -49 \times 100 = -4900 J$
- c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

$$P = \frac{W}{t} = 4900 / 10.54 = 465 W$$

- 24. h = 10 m
 - flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power P =
$$\frac{\text{mgh}}{\text{t}}$$
 = (0.5) × 9.8 × 10 = 49 W

- So, horse power (h.p) $P/746 = 49/746 = 6.6 \times 10^{-2}$ hp
- 25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 secTotal work done = $\frac{1}{2}mv^2 + mgh = (\frac{1}{2}) \times (0.2) \times 9 + (0.2) \times (9.8) \times (1.5) = 3.84 J$

h.p. used =
$$\frac{3.84}{746}$$
 = 5.14 × 10⁻³

- 26. m = 200 kg, s = 12m, t = 1 min = 60 sec
 - So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work] = 2000 × 10 × 12 = 240000 J

So, power p =
$$\frac{W}{t} = \frac{240000}{60} = 4000$$
 watt

$$h.p = \frac{4000}{746} = 5.3 \text{ hp.}$$

27. The specification given by the company are

$$U = 0$$
, $m = 95 \text{ kg}$, $P_m = 3.5 \text{ hp}$

$$V_m = 60 \text{ km/h} = 50/3 \text{ m/sec}$$
 $t_m = 5 \text{ sec}$

So, the maximum acceleration that can be produced is given by,

$$a = \frac{(50/3) - 0}{5} = \frac{10}{3}$$

So, the driving force is given by

$$F = ma = 95 \times \frac{10}{3} = \frac{950}{3} N$$

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

$$v = \frac{p}{F} \Rightarrow v = \frac{3.5 \times 746 \times 5}{950} = 8.2 \text{ m/sec.}$$

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30 kg, v = 40 cm/sec = 0.4 m/sec s = 2 m

From the free body diagram, the force given by the chain is,

F = (ma - mg) = m(a - g) [where a = acceleration of the block]

$$a = \frac{(v2 \text{ u2})}{2s} = \frac{0.16}{0.4} = 0.04 \text{ m/sec}^2$$

So, work done W = Fs $\cos \theta = m(a - g) s \cos \theta$

$$\Rightarrow$$
 W = 30 (0.04 - 9.8) × 2 \Rightarrow W = -585.5 \Rightarrow W = -586 J.

So,
$$W = -586 J$$

29. Given, T = 19 N

From the freebody diagrams,

$$T - 2 mg + 2 ma = 0$$
 ...

$$T - mg - ma = 0$$

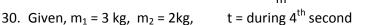
From, Equation (i) & (ii) T = 4ma
$$\Rightarrow$$
 a = $\frac{T}{4m} \Rightarrow$ A = $\frac{16}{4m} = \frac{4}{m}$ m/s².

Now,
$$S = ut + \frac{1}{2} at^2$$

$$\Rightarrow$$
 S = $\frac{1}{2} \times \frac{4}{m} \times 1 \Rightarrow$ S = $\frac{2}{m}$ m [because u=0]

Net mass = 2m - m = m

Decrease in P.E. = mgh
$$\Rightarrow$$
 P.E. = m × g × $\frac{2}{m}$ \Rightarrow P.E. = 9.8 × 2 \Rightarrow P.E. = 19.6 J



From the freebody diagram
$$T - 3g + 3a = 0$$
 ...(i)

$$T - 2g - 2a = 0$$
 ..(ii)

Equation (i) & (ii), we get
$$3g - 3a = 2g + 2a \Rightarrow a = \frac{g}{5}$$
 m/sec²

Distance travelled in 4th sec is given by

$$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2\times4-1) = \frac{7g}{10} = \frac{7\times9.8}{10} \text{ m}$$

Net mass 'm' =
$$m_1 - m_2 = 3 - 2 = 1 \text{kg}$$

So, decrease in P.E. = mgh =
$$1 \times 9.8 \times \frac{7}{10} \times 9.8 = 67.2 = 67 \text{ J}$$

31.
$$m_1 = 4kg$$
, $m_2 = 1kg$, $V_2 = 0.3m/sec$ $V_1 = 2 \times (0.3) = 0.6$ m/sec

 $(v_1 = 2x_2 \text{ m this system})$

h = 1m = height descent by 1kg block

$$s = 2 \times 1 = 2m$$
 distance travelled by 4kg block

$$u = 0$$

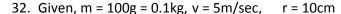
Applying change in K.E. = work done (for the system)

$$[(1/2)m_1v_1^2 + (1/2) m_2v_m^2] - 0 = (-\mu R)S + m_2g$$
 [R = 4g = 40 N]

$$\Rightarrow \frac{1}{2} \times 4 \times (0.36) \times \frac{1}{2} \times 1 \times (0.09) = -\mu \times 40 \times 2 + 1 \times 40 \times 1$$

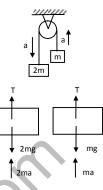
$$\Rightarrow$$
 0.72 + 0.045 = -80 μ + 10

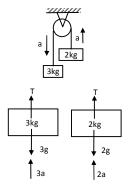
$$\Rightarrow \mu = \frac{9.235}{80} = 0.12$$

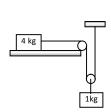


$$(1/2 \text{ mv}^2 + \text{mgh}) - 0$$

$$\Rightarrow$$
 W = ½ mv² + mgh – 0 = ½ × (0.1) × 25 + (0.1) × 10 × (0.2) [h = 2r = 0.2m]









$$\Rightarrow$$
 W = 1.25 – 0.2 \Rightarrow W = 1.45 J

So, the work done by the tube on the body is

$$W_t = -1.45 J$$

33. m = 1400 kg, v = 54 km/h = 15 m/sec,

$$h = 10m$$

Work done = (total K.E.) - total P.E.

=
$$0 + \frac{1}{2} \text{ mv}^2 - \text{mgh} = \frac{1}{2} \times 1400 \times (15)^2 - 1400 \times 9.8 \times 10 = 157500 - 137200 = 203008$$

So, work done against friction, W_t = 20300 J



34. m = 200g = 0.2kg, s = 10m,

$$h = 3.2m$$
,

$$g = 10 \text{ m/sec}^2$$

- a) Work done W = mgh = $0.2 \times 10 \times 3.2 = 6.4 \text{ J}$
- b) Work done to slide the block up the incline

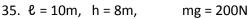
w = (mg sin
$$\theta$$
) = (0.2) × 10 × $\frac{3.2}{10}$ × 10 = 6.4 J

c) Let, the velocity be v when falls on the ground vertically,

$$\frac{1}{2} \text{ mv}^2 - 0 = 6.4 \text{J} \Rightarrow \text{v} = 8 \text{ m/s}$$

d) Let V be the velocity when reaches the ground by liding

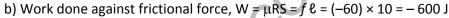
$$\frac{1}{2} \text{ mV}^2 - 0 = 6.4 \text{ J} \Rightarrow \text{V} = 8\text{m/sec}$$



$$f = 200 \times \frac{3}{10} = 60$$
N



a) Work done by the ladder on the boy is zero when the boy is going up because the work is done by the boy himself.



c) Work done by the forces inside the boy is

$$W_b = (mg \sin\theta) \times 10 = 200 \times \frac{8}{10} \times 10 = 1600 \text{ J}$$



36. H = 1m, h = 0.5m

Applying law of conservation of Energy for point A & B

mgH =
$$\frac{1}{2}$$
 mv² + mgh \Rightarrow g = (1/2) v² + 0.5g \Rightarrow v² 2(g - 0.59) = g \Rightarrow v = \sqrt{g} = 3.1 m/s

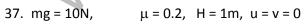
After point B the body exhibits projectile motion for which

$$\theta = 0^{\circ}, \quad v = -0.5$$

So,
$$-0.5 = (u \sin \theta) t - (1/2) gt^2 \Rightarrow 0.5 = 4.9 t^2 \Rightarrow t = 0.31 sec.$$

So,
$$x = (4 \cos \theta) t = 3.1 \times 3.1 = 1m$$
.

So, the particle will hit the ground at a horizontal distance in from B.



change in P.E. = work done.

Increase in K.E.

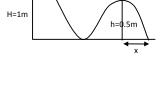
$$\Rightarrow$$
 w = mgh = 10 × 1 = 10 J

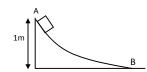
Again, on the horizontal surface the fictional force

$$F = \mu R = \mu mg = 0.2 \times 10 = 2 N$$

So, the K.E. is used to overcome friction

$$\Rightarrow$$
 S = $\frac{W}{F}$ = $\frac{10J}{2N}$ = 5m





38. Let 'dx' be the length of an element at a distance \times from the table

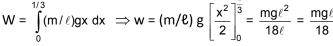
mass of 'dx' length = (m/ℓ) dx

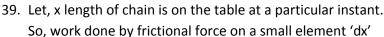
Work done to put dx part back on the table

$$W = (m/\ell) dx g(x)$$

So, total work done to put ℓ/3 part back on the table

$$W = \int_{0}^{1/3} (m/\ell) gx \, dx \implies w = (m/\ell) g \left[\frac{x^2}{2} \right]_{0}^{\frac{\ell}{3}} = \frac{mg\ell^2}{18\ell} = \frac{mg\ell}{18}$$





$$dW_f = \mu Rx = \mu \left(\frac{M}{L}dx\right)gx$$

[where
$$dx = \frac{M}{I} dx$$
]

Total work don by friction,

$$W_f = \int\limits_{2L/3}^0 \! \mu \frac{M}{L} gx \ dx$$

:.
$$W_f = \mu \frac{m}{L} g \left[\frac{x^2}{2} \right]_{2L/3}^0 = \mu \frac{M}{L} \left[\frac{4L^2}{18} \right] = 2\mu Mg L/9$$

40. Given, m = 1kg, H = 1m, h = 0.8m

Here, work done by friction = change in P.E. [as the body comes to rest]

$$\Rightarrow$$
 W_f = mgh – mgH

$$= mg (h - H)$$

$$= 1 \times 10 (0.8 - 1) = -2J$$

41. m = 5kg, x = 10cm = 0.1m,

h =?
$$G = 10 \text{m/sec}^2$$

S0,
$$k = \frac{mg}{x} = \frac{50}{0.1} = 500 \text{ N/m}$$

Total energy just after the blow $E = \frac{1}{2} \text{ mv}^2 + \frac{1}{2} \text{ kx}^2$...(i)

Total energy a a height $h = \frac{1}{2} k (h - x)^2 + mgh$...(ii)

$$\frac{1}{2}$$
 mv² + $\frac{1}{2}$ kx² = $\frac{1}{2}$ k (h – x)² + mgh

On, solving we can get,

$$H = 0.2 m = 20 cm$$

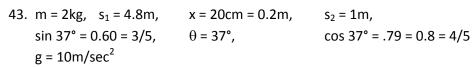
42. m = 250 g = 0.250 kg,

$$k = 100 \text{ N/m}, \qquad m = 10 \text{ cm} = 0.1 \text{m}$$

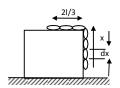
$$g = 10 \text{ m/sec}^2$$

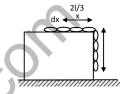
Applying law of conservation of energy

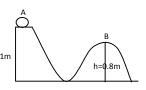
$$\frac{1}{2} kx^2 = mgh \Rightarrow h = \frac{1}{2} \left(\frac{kx^2}{mg} \right) = \frac{100 \times (0.1)^2}{2 \times 0.25 \times 10} = 0.2 \text{ m} = 20 \text{ cm}$$

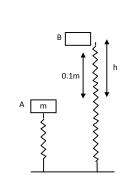


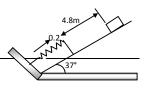
Applying work – Energy principle for downward motion of the body













$$0 - 0 = \text{mg sin } 37^{\circ} \times 5 - \mu R \times 5 - \frac{1}{2} \text{ kx}^{2}$$

$$\Rightarrow$$
 20 × (0.60) × 1 – μ × 20 × (0.80) × 1 + ½ k (0.2)² = 0

$$\Rightarrow$$
 60 - 80 μ - 0.02k = 0 \Rightarrow 80 μ + 0.02k = 60 ...(i)

Similarly, for the upward motion of the body the equation is

$$0 - 0 = (-mg \sin 37^{\circ}) \times 1 - \mu R \times 1 + \frac{1}{2} k (0.2)^{2}$$

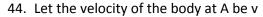
$$\Rightarrow$$
 -20 × (0.60) × 1 - μ ×20 × (0.80) × 1 + $\frac{1}{2}$ k (0.2)² = 0

$$\Rightarrow$$
 -12 - 16 μ + 0.02 K = 0 ...(ii)

Adding equation (i) & equation (ii), we get 96 μ = 48

$$\Rightarrow \mu = 0.5$$

Now putting the value of μ in equation (i) K = 1000N/m

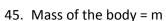


So, the velocity of the body at B is v/2

Energy at point A = Energy at point B

So,
$$\frac{1}{2}$$
 mv_A² = $\frac{1}{2}$ mv_B² + $\frac{1}{2}$ kx²⁺

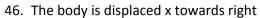
$$\Rightarrow \frac{1}{2} kx^2 = \frac{1}{2} mv_A^2 - \frac{1}{2} mv_B^2 \Rightarrow kx^2 = m \left(v_A^{2+-} v_B^2 \right) \Rightarrow kx^2 = m \left(v^2 - \frac{v^2}{4} \right) \Rightarrow k = \frac{3mv^2}{3x^2}$$



Let the elongation be x

So,
$$\frac{1}{2}$$
 kx² = mgx

$$\Rightarrow$$
 x = 2mg / k



Let the velocity of the body be v at its mean position

Applying law of conservation of energy

$$\frac{1}{2}$$
 mv² = $\frac{1}{2}$ k₁x² + $\frac{1}{2}$ k₂x² \Rightarrow mv² = x² (k₁ + k₂) \Rightarrow v² = $\frac{x^2(k_1 + k_2)}{m}$

$$\Rightarrow$$
 v = $x\sqrt{\frac{k_1+k_2}{m}}$

47. Let the compression be x

According to law of conservation of energy

$$\frac{1}{2}$$
 mv² = $\frac{1}{2}$ kx² \Rightarrow x² = mv² / k \Rightarrow x = v $\sqrt{\frac{m}{k}}$



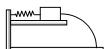
b) No. It will be in the opposite direction and magnitude will be less due to loss in spring.

48.
$$m = 100g = 0.1kg$$
, $x = 5c$

$$x = 5cm = 0.05m$$
, $k = 100N/m$

when the body leaves the spring, let the velocity be v

$$\frac{1}{2} \text{ mv}^2 = \frac{1}{2} \text{ kx}^2 \Rightarrow \text{v} = \frac{1}{2} \sqrt{\frac{100}{0.1}} = 1.58 \text{ m/sec}$$



For the projectile motion, $\theta = 0^{\circ}$, Y = -2

Now,
$$y = (u \sin \theta)t - \frac{1}{2}gt^2$$

$$\Rightarrow$$
 -2 = (-1/2) \times 9.8 \times t² \Rightarrow t = 0.63 sec.

So,
$$x = (u \cos \theta) t \Rightarrow 1.58 \times 0.63 = 1m$$



49. Let the velocity of the body at A is 'V' for minimum velocity given at A velocity of the body at point B is zero.

Applying law of conservation of energy at A & B

$$\frac{1}{2} \text{ mv}^2 = \text{mg (2\ell)} \Rightarrow \text{v} = \sqrt{(4g\ell)} = 2\sqrt{g\ell}$$

50. m = 320g = 0.32kg

k = 40N/m

h = 40cm = 0.4m

 $g = 10 \text{ m/s}^2$

From the free body diagram,

 $kx \cos \theta = mg$

(when the block breaks off R = 0)

 \Rightarrow cos θ = mg/kx

So,
$$\frac{0.4}{0.4 + x} = \frac{3.2}{40 \times x} \Rightarrow 16x = 3.2x + 1.28 \Rightarrow x = 0.1 \text{ m}$$

S0, s = AB =
$$\sqrt{(h+x)^2 - h^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3 \text{ m}$$

Let the velocity of the body at B be v

Charge in K.E. = work done (for the system)

$$(1/2 \text{ mv}^2 + \frac{1}{2} \text{ mv}^2) = -1/2 \text{ kx}^2 + \text{mgs}$$

$$\Rightarrow$$
 (0.32) \times v² = -(1/2) \times 40 \times (0.1)² + 0.32 \times 10 \times (0.3) \Rightarrow v = 1.5 m/s

51. θ = 37°; I = h = natural length

Let the velocity when the spring is vertical be 'v'

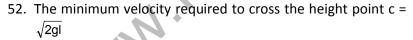
$$Cos 37^{\circ} = BC/AC = 0.8 = 4/5$$

$$Ac = (h + x) = 5h/4 \text{ (because BC = h)}$$

So,
$$x = (5h/4) - h = h/4$$

Applying work energy principle $\frac{1}{2} kx^2 = \frac{1}{2} mv^2$

$$\Rightarrow$$
 v = x $\sqrt{(k/m)} = \frac{h}{4} \sqrt{\frac{k}{m}}$



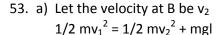
Let the rod released from a height h.

Total energy at A = total energy at B

$$mgh = 1/2 \text{ mv}^2$$
; $mgh = 1/2 \text{ m (2gl)}$

[Because v = required velocity at B such that the block makes a complete circle. [Refer Q - 49]

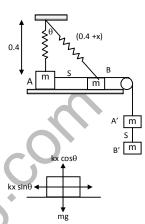
So,
$$h = I$$
.

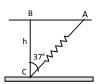


$$\Rightarrow$$
 1/2 m (10 gl) = 1/2 mv₂² + mgl

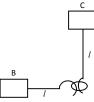
$$v_2^2 = 8 gl$$

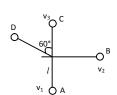
So, the tension in the string at horizontal position

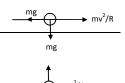












$$T = \frac{mv^2}{R} = \frac{m8gI}{I} = 8 \text{ mg}$$

b) Let the velocity at C be V₃

$$1/2 \text{ mv}_1^2 = 1/2 \text{ mv}_3^2 + \text{mg (2I)}$$

$$\Rightarrow$$
 1/2 m (log l) = 1/2 mv₃² + 2mgl

$$\Rightarrow$$
 $v_3^2 = 6 \text{ mgl}$

So, the tension in the string is given by

$$T_c = \frac{mv^2}{I} - mg = \frac{6 \text{ glm}}{I} \text{ mg} = 5 \text{ mg}$$

c) Let the velocity at point D be v₄

Again,
$$1/2 \text{ mv}_1^2 = 1/2 \text{ mv}_4^2 + \text{mgh}$$

$$1/2 \times m \times (10 \text{ gl}) = 1.2 \text{ mv}_4^2 + \text{mgl} (1 + \cos 60^\circ)$$

$$\Rightarrow$$
 $v_4^2 = 7 gI$

So, the tension in the string is

$$T_D = (mv^2/I) - mg \cos 60^\circ$$

=
$$m(7 gl)/l - l - 0.5 mg \Rightarrow 7 mg - 0.5 mg = 6.5 mg$$
.

54. From the figure, $\cos \theta = AC/AB$

$$\Rightarrow$$
 AC = AB cos $\theta \Rightarrow$ (0.5) × (0.8) = 0.4.

So,
$$CD = (0.5) - (0.4) = (0.1) m$$

Energy at D = energy at B

$$1/2 \text{ mv}^2 = \text{mg (CD)}$$

$$v^2 = 2 \times 10 \times (0.1) = 2$$

So, the tension is given by,

T =
$$\frac{mv^2}{r}$$
 + mg = (0.1) $\left(\frac{2}{0.5}$ + 10 $\right)$ = 1.4 N.



As shown in the figure, $mv^2 / R = mg$

$$\Rightarrow$$
 v² = gR ...(2

$$1/2 \text{ kx}^2 = \frac{\text{mgR} + 2\text{mgR}}{2}$$
 [because $v^2 = gR$]

$$\Rightarrow$$
 x² = 3mgR/k \Rightarrow x = $\sqrt{(3mgR)/k}$.



$$1/2 \text{ mv}^2 - 1/2 \text{ mu}^2 = -\text{mgh}$$

$$v^2 = u^2 - 2g(I + I\cos\theta)$$

$$\Rightarrow$$
 v² = 3gl – 2gl (1 + cos θ) ...(1)

Again,

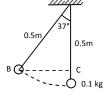
$$mv^2/I = mg \cos \theta$$

$$v^2 = \lg \cos \theta$$

From equation (1) and (2), we get

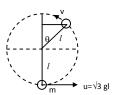
$$3gl - 2gl - 2gl \cos \theta = gl \cos \theta$$













$$3 \cos \theta = 1 \Rightarrow \cos \theta = 1/3$$

$$\theta = \cos^{-1}(1/3)$$

So, angle rotated before the string becomes slack

$$= 180^{\circ} - \cos^{-1}(1/3) = \cos^{-1}(-1/3)$$

57.
$$I = 1.5 \text{ m}$$
; $u = \sqrt{57} \text{ m/sec}$.

a) mg cos
$$\theta = mv^2 / I$$

$$v^2 = \lg \cos \theta$$

change in K.E. = work done

$$1/2 \text{ mv}^2 - 1/2 \text{ mu}^2 = \text{mgh}$$

$$\Rightarrow$$
 v² - 57 = -2 × 1.5 g (1 + cos θ)...(2)

$$\Rightarrow$$
 v² = 57 – 3g(1 + cos θ)

Putting the value of v from equation (1)

15 cos θ = 57 – 3g (1 + cos θ)
$$\Rightarrow$$
 15 cos θ = 57 – 30 – 30 cos θ

$$\Rightarrow$$
 45 cos θ = 27 \Rightarrow cos θ = 3/5.

$$\Rightarrow \theta = \cos^{-1}(3/5) = 53^{\circ}$$

b)
$$v = \sqrt{57 - 3g(1 + \cos \theta)}$$
 from equation (2)

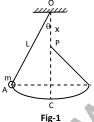
$$= \sqrt{9} = 3 \text{ m/sec.}$$

c) As the string becomes slack at point B, the particle will start making projectile motion.

$$H = OE + DC = 1.5 \cos \theta + \frac{u^2 \sin^2 \theta}{2q}$$

=
$$(1.5) \times (3/5) + \frac{9 \times (0.8)^2}{2 \times 10} = 1.2 \text{ m}.$$

58.



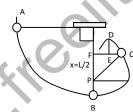
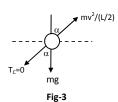


Fig-2



a) When the bob has an initial height less than the peg and then released from rest (figure 1), let body travels from A to B.

Since, Total energy at A = Total energy at B

∴
$$(K.E)_A = (PE)_A = (KE)_B + (PE)_B$$

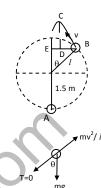
$$(PE)_A = (PE)_B$$
 [because, $(KE)_A = (KE)_B = 0$]

So, the maximum height reached by the bob is equal to initial height.

b) When the pendulum is released with $\theta = 90^{\circ}$ and x = L/2, (figure 2) the path of the particle is shown in the figure 2.

At point C, the string will become slack and so the particle will start making projectile motion. (Refer Q.No. 56)

$$(1/2)$$
mv_c² - 0 = mg (L/2) (1 - cos α)



because, distance between A nd C in the vertical direction is L/2 (1 – $\cos \alpha$)

$$\Rightarrow$$
 $v_c^2 = gL(1 - \cos \theta)$...(1)

Again, form the freebody diagram (fig - 3)

$$\frac{\text{mv}_c^2}{1/2} = \text{mg cos } \alpha \text{ {because T}_c = 0}$$

So,
$$V_c^2 = \frac{gL}{2}\cos\alpha$$
 ...(2)

From Eqn.(1) and equn (2),

$$gL (1 - \cos \alpha) = \frac{gL}{2} \cos \alpha$$

$$\Rightarrow$$
 1 – cos α = 1/2 cos α

$$\Rightarrow$$
 3/2 cos α = 1 \Rightarrow cos α = 2/3 ...(3)

To find highest position C, before the string becomes slack

BF =
$$\frac{L}{2} + \frac{L}{2}\cos\theta = \frac{L}{2} + \frac{L}{2} \times \frac{2}{3} = L\left(\frac{1}{2} + \frac{1}{3}\right)$$

So,
$$BF = (5L/6)$$

c) If the particle has to complete a vertical circle, at the point C.

$$\frac{mv_c^2}{(L-x)} = mg$$

$$\Rightarrow$$
 $v_c^2 = g(L - x)$...(1)

Again, applying energy principle between A and C,

$$1/2 \text{ mv}_c^2 - 0 = \text{mg (OC)}$$

$$\Rightarrow$$
 1/2 $v_c^2 = mg [L - 2(L - x)] = mg (2x - L)$

$$\Rightarrow$$
 $v_c^2 = 2g(2x - L)$...(2)

From equn. (1) and equn (2)

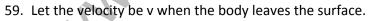
$$g(L-x) = 2g(2x-L)$$

$$\Rightarrow$$
 L - x = 4x - 2L

$$\Rightarrow$$
 5x = 3L

$$\therefore \frac{x}{L} = \frac{3}{5} = 0.6$$

So, the rates (x/L) should be 0.6



From the freebody diagram,

$$\frac{\text{mv}^2}{\text{R}}$$
 = mg cos θ [Because normal reaction]

$$v^2 = Rg \cos \theta \qquad ..(1)$$

Again, form work-energy principle,

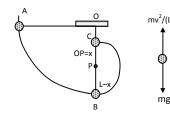
Change in K.E. = work done

$$\Rightarrow$$
 1/2 mv² – 0 = mg(R – R cos θ)

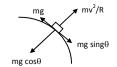
$$\Rightarrow$$
 v² = 2gR (1 - cos θ) ...(2)

From (1) and (2)

Rg cos
$$\theta$$
 = 2gR (1 – cos θ)







$$3gR \cos \theta = 2gR$$

$$\cos \theta = 2/3$$

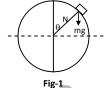
$$\theta = \cos^{-1}(2/3)$$

60. a) When the particle is released from rest (fig-1), the centrifugal force is zero.

N force is zero = mg cos
$$\theta$$

= mg cos 30° =
$$\frac{\sqrt{3}\text{mg}}{2}$$





So,
$$\frac{mv^2}{R}$$
 mg cos θ

$$\Rightarrow$$
 v² = Rg cos θ ...(1)

Again,
$$\frac{1}{2}$$
 mv² = mgR (cos 30° – cos θ)

$$\Rightarrow$$
 v² = 2Rg $\left(\frac{\sqrt{3}}{2} - \cos \theta\right)$..(2)

From equn. (1) and equn. (2)

Rg cos
$$\theta = \sqrt{3}$$
 Rg – 2Rg cos θ

$$\Rightarrow$$
 3 cos $\theta = \sqrt{3}$

$$\Rightarrow \cos \theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

So, the distance travelled by the particle before leaving contact,

$$\ell = R(\theta - \pi/6)$$
 [because 30° = $\pi/6$]

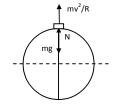
putting the value of
$$\theta$$
, we get $\ell = 0.43R$

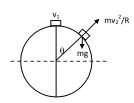


horizontal speed = v

From the free body diagram, (fig-1)

N = Normal force = mg -
$$\frac{mv^2}{R}$$





b) When the particle is given maximum velocity so that the centrifugal force balances the weight, the particle does not slip on the sphere.

$$\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$$

c) If the body is given velocity v₁

$$V_{1} = \sqrt{gR}/2$$

$$v_1^2 - gR / 4$$

Let the velocity be v_2 when it leaves contact with the surface, (fig-2)

So,
$$\frac{mv^2}{R}$$
 = mg cos θ

$$\Rightarrow v_2^2 = Rg \cos \theta$$
 ..(1)

Again,
$$1/2 \text{ mv}_2^2 - 1/2 \text{ mv}_1^2 = \text{mgR} (1 - \cos \theta)$$

$$\Rightarrow$$
 $v_2^2 = v_1^2 + 2gR (1 - cos θ)$...(2)

Rg cos
$$\theta$$
 = (Rg/4) + 2gR (1 – cos θ)

$$\Rightarrow$$
 cos θ = (1/4) + 2 - 2 cos θ

$$\Rightarrow$$
 3 cos θ = 9/4

$$\Rightarrow$$
 cos θ = 3/4

$$\Rightarrow \theta = \cos^{-1} (3/4)$$

62. a) Net force on the particle between A & B, F = mg sin θ

work done to reach B, W = FS = mg sin θ &

Again, work done to reach B to C = mgh = mg R $(1 - \cos \theta)$

So, Total workdone = $mg[\ell \sin \theta + R(1 - \cos \theta)]$

Now, change in K.E. = work done

$$\Rightarrow$$
 1/2 mv₀² = mg [ℓ sin θ + R (1 – cos θ)

$$\Rightarrow$$
 v_o = $\sqrt{2g(R(1-\cos\theta)+\ell\sin\theta)}$

b) When the block is projected at a speed 2vo.

Let the velocity at C will be V_c.

Applying energy principle,

$$1/2 \text{ mv}_c^2 - 1/2 \text{ m} (2v_0)^2 = -\text{mg} [\ell \sin \theta + R(1 - \cos \theta)]$$

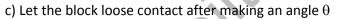
$$\Rightarrow$$
 $v_c^2 = 4v_o - 2g \left[e \sin \theta + R(1 - \cos \theta) \right]$

4.2g
$$[\ell \sin \theta + R(1 - \cos \theta)] - 2g [\ell \sin \theta + R(1 - \cos \theta)]$$

= 6g [
$$\ell$$
sin θ + R(1 – cos θ)]

So, force acting on the body,

$$\Rightarrow N = \frac{mv_c^2}{R} = 6mg [(\ell/R) \sin \theta + 1 - \cos \theta]$$



$$\frac{\text{mv}^2}{\text{R}}$$
 = mg cos $\theta \Rightarrow \text{v}^2$ = Rg cos θ ...(1)

Again,
$$1/2 \text{ mv}^2 = \text{mg} (R - R \cos \theta) \Rightarrow v^2 = 2gR (1 - \cos \theta)$$
 ...(2)......(?)

From (1) and (2)
$$\cos \theta = 2/3 \Rightarrow \theta = \cos^{-1}(2/3)$$

63. Let us consider a small element which makes angle ' $d\theta$ ' at the centre.

∴ dm = (m/
$$\ell$$
)Rd θ

a) Gravitational potential energy of 'dm' with respect to centre of the sphere

= (dm)g R cos
$$\theta$$

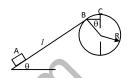
=
$$(mg/\ell) R\cos \theta d\theta$$

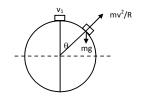
So, Total G.P.E. =
$$\int_0^{\ell/r} \frac{\text{mgR}^2}{\ell} \cos \theta \, d\theta$$
 [$\alpha = (\ell/R)$](angle subtended by the chain at the

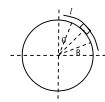
centre).....

$$= \frac{mR^2g}{\ell} [\sin \theta] (\ell/R) = \frac{mRg}{\ell} \sin (\ell/R)$$

- b) When the chain is released from rest and slides down through an angle θ , the K.E. of the chain is given
- K.E. = Change in potential energy.







$$= \frac{mR^2g}{\ell} \sin(\ell/R) - m \int \frac{gR^2}{\ell} \cos \theta d\theta...$$
?

$$= \frac{mR^2g}{\ell} \left[\sin (\ell/R) + \sin \theta - \sin \{\theta + (\ell/R)\} \right]$$

c) Since, K.E. =
$$1/2 \text{ mv}^2 = \frac{\text{mR}^2 \text{g}}{\ell} [\sin (\ell/R) + \sin \theta - \sin \{\theta + (\ell/R)\}]$$

Taking derivative of both sides with respect to 't'

$$(1/2) \times 2v \times \frac{dv}{dt} = \frac{R^2g}{\ell} \left[\cos \theta \times \frac{d\theta}{dt} - \cos (\theta + \ell/R) \frac{d\theta}{dt} \right]$$

$$\therefore (R \frac{d\theta}{dt}) \frac{dv}{dt} = \frac{R^2g}{\ell} \times \frac{d\theta}{dt} [\cos \theta - \cos (\theta + (\ell/R))]$$

When the chain starts sliding down, θ = 0.

So,
$$\frac{dv}{dt} = \frac{Rg}{\ell} [1 - \cos(\ell/R)]$$

64. Let the sphere move towards left with an acceleration 'a

Let m = mass of the particle

The particle 'm' will also experience the inertia due to acceleration 'a' as it is on the sphere. It will also experience the tangential inertia force (m (dv/dt)) and centrifugal force (mv^2/R).

$$m\frac{dv}{dt} = \text{ ma cos } \theta + \text{ mg sin } \theta \Rightarrow \text{ mv } \frac{dv}{dt} = \text{ ma cos } \theta \left(R\frac{d\theta}{dt}\right) + \text{ mg sin } \theta$$

$$\frac{\partial}{\partial t} = \text{ma cos } \theta + \text{mg sin } \theta \Rightarrow \text{mv } \frac{\partial}{\partial t} = \text{ma cos } \theta \left(R \frac{\partial}{\partial t} \right) + \text{mg sin } \theta$$

Because,
$$v = R \frac{d\theta}{dt}$$

$$\Rightarrow$$
 vd v = a R cos θ d θ + gR sin θ d θ

Integrating both sides we get,

$$\frac{v^2}{2} = a R \sin \theta - gR \cos \theta + C$$

Given that, at θ = 0, v = 0 , So, C = gR

So,
$$\frac{v^2}{2}$$
 = a R sin θ – g R cos θ + g R

$$\therefore v^2 = 2R (a \sin \theta + g - g \cos \theta) \Rightarrow v = [2R (a \sin \theta + g - g \cos \theta)]^{1/2}$$

